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SHORTER COMMUNICATIONS

BUBBLE GROWTH AND COLLAPSE IN NARROW TUBES WITH NONUNIFORM INITIAL TEMPERATURE PROFILES

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NOMENCLATURE

A, cross-sectional area of channel;
C, heat capacity;
T_C, temperature of fluid within cold jacket;
T_H, temperature of fluid within hot jacket;
$t_0(x)$, time at which liquid film is exposed to vapor space;
$t'_0(x)$, time at which cold liquid film is exposed to hot liquid;
v, velocity of liquid slug;
x, axial position along length of test section.

Subscripts
i, reference to free surface;
s, liquid slug.

Greek letters
α, void fraction; also thermal diffusivity;
σ, distance within liquid film normal to vapor–liquid surface; also surface tension;
τ, time;
ϕ, temperature within vapor slug.

IN A PREVIOUS paper [1], slug expulsion of Freon-113 initially at a uniform temperature by rapid depressurization of a vertical tube, was discussed. In the present work, data are presented for both expulsion and reentry of Freon-113 with nonuniform initial temperature, and the previous analysis is extended to take into account both the effect of the film thickness and the nonuniform initial condition. The experimental facility has been described previously [1, 2]. In the present work, the test sections consisted of a vertical glass tube with inside diameters of 0·44–0·7 cm, a heated length of about 12 in., and a cold-zone length of 24 in. surmounted by a plenum chamber. Bubble growth was initiated either by a wire electrode or by puncturing an aluminum diaphragm connecting to the vacuum reservoir.

In computing the total energy balance on the growing bubble [1], the gradient at the film–vapor interface was here evaluated from the solution to the heat equation for the liquid film, assuming the existence of a thin thermal boundary layer within the film, whose thickness as a function of exposure time and position along the vapor space is taken into account. This leads to

$$\left.\frac{\mathrm{d}T[\sigma, t, t_0(x)]}{\mathrm{d}\sigma}\right|_{\sigma=0} = \frac{T_H - \phi[t_0(x)]}{\sqrt{\{\pi\alpha[t - t_0(x)]\}}} \int_0^{t-t_0} \frac{1}{\sqrt{\{\pi\alpha[t - t_0(x) - \tau]\}}} \frac{\mathrm{d}\phi(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau \qquad (1)$$

within the hot zone.

Within the cold zone, it was assumed that, as the vapor space began to grow within the hot zone, a hot slug of liquid was pushed into the cold zone and began to transfer energy to the cold film remaining on the walls of the channel. The amount of cold film exposed during each time increment was equal to the corresponding growth in the vapor space during the same time increment. The time at which each increment was exposed to the hot liquid was recorded. The heat equation and corresponding boundary conditions for the interface between hot liquid and cold film lead to the following expression for the surface heat flux of an element at the height x, initially at temperature T_C, exposed at time t'_0 to the hot liquid slug initially at temperature T_H.

$$T[\sigma, t, t'_0(x)] = T_H - (T_H - T_C)\,\mathrm{erf}\left[\frac{-\sigma}{2\sqrt{\{\alpha[t - t'_0(x)]\}}}\right]. \qquad (2)$$

As each increment of film within the cold zone is exposed to vapor, the length of time which that increment has been exposed to the hot liquid slug is recorded. Equation (2) is then inserted into equation (1) to yield

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$$\left.\frac{\mathrm{d}T[\sigma, t, t_0(x), t_0'(x)]}{\mathrm{d}\sigma}\right|_{\sigma=0} = -\int_0^{t-t_0(x)} \frac{1}{\sqrt{\{\pi\alpha[t - t_0(x) - \tau]\}}} \times \frac{\mathrm{d}\phi(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau - \frac{\phi(0)}{\sqrt{\{\pi\alpha[t - t_0(x)]\}}} + \frac{T_H}{\sqrt{\{\pi\alpha[t - t_0(x)]\}}} - (T_H - T_C)\int_0^{\infty} \frac{\sigma}{\sqrt{\{\pi\alpha[t - t_0(x)]^3\}}} \exp\left[\frac{-\sigma^2}{4\alpha[t - t_0(x)]}\right] \mathrm{erf}\left[\frac{\sigma}{2\sqrt{\{\alpha[t_0(x) - t_0'(x)]\}}}\right]\mathrm{d}\sigma \quad (3)$$

within the cold zone. During reentry, as each increment of film was covered by the reentering slug, the contribution of that increment of film was no longer considered as an energy source of the vapor. The numerical solution to equations (1)–(3) proceeds as discussed in [1].

EXPERIMENTAL RESULTS

In the analysis of the experiments performed in [1], the void fraction was assumed to be equal to one. To account for the effect of film thickness in the coupled solution of the energy and momentum equations, an accurate value for the void fraction had to be determined. The technique described in [3] was employed.

If it is assumed that the liquid above the bubble is incompressible and that there is no drainage around the nose of the bubble, the time rate of change of the volume of the bubble equals the time rate of change of the volume displaced by the free surface. Therefore,

$$v_i A_i = v_s A_s, \alpha \equiv \frac{A_s}{A_i} = \frac{v_i}{v_s} \quad (4)$$

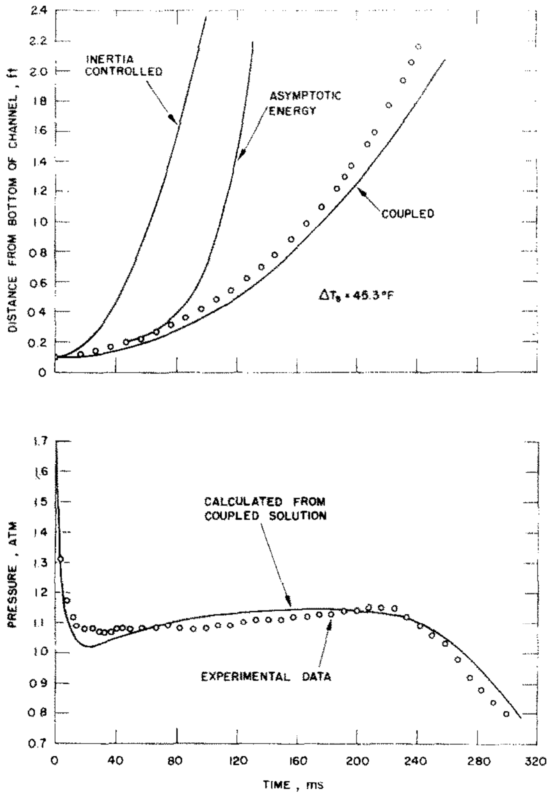

FIG. 2. Distance of upper liquid–vapor interface from bottom of channel and pressure within the vapor space vs. time for a slug expulsion initiated by a wire electrode.

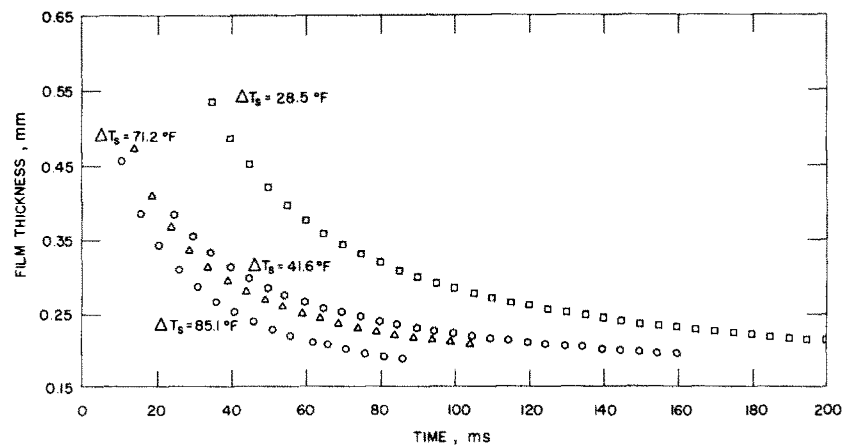

FIG. 1. Liquid film thickness vs. time for a slug expulsion initiated by a wire electrode.

where the instantaneous local void fraction has been expressed as the ratio of the free surface velocity to the bubble velocity.

The displacement of both the upper and lower interfaces as a function of time was measured by means of two synchronized high-speed cameras.

The experimental data were then fitted to a second-order polynomial by a least-square routine, differentiated, and substituted into equation (4). Figure 1 illustrates the average film thickness as a function of time for various initial superheats. In each case, the liquid fraction approached an asymptotic value of approximately 0·15. Thus, this would be used to estimate the liquid film thickness over the major portion of each run. The ratio of the bubble velocity to the free surface velocity also approaches an asymptotic value of approximately 1·20 [3]. This result is quite similar to the ratio of the maximum-to-average free-stream velocity in the liquid above the bubble for fully-developed turbulent flow. This has been noted in steady multi-bubble two-phase slug flow [4].

Figure 2 illustrates the agreement between the coupled solution and data obtained from expulsions initiated by the wire electrode. The coupled solution accounts both for an average void fraction and for the heating of the cold wall film by the hot liquid slug pushed into the cold zone. To account for the additional energy supplied to the vapor space by the wire electrode, the instantaneous power generated in the entire electrode was calculated, and the fraction of the wire electrode exposed to liquid was used to determine the fraction of the total instantaneous power that was supplied to the bubble. This energy source was applied during the time current was allowed to flow through the electrode.

The assumption of a thin thermal boundary layer was justified in [1] based upon the film thickness measurements of Peppler *et al.* [5]. For a Freon expulsion of 400 ms in duration, the thermal boundary layer thickness will be 0·13 mm. If this thickness is compared with the asymptotic film thickness shown in Fig. 1 of 0·20 mm, the thin thermal boundary layer assumption appears valid.

Figures 3 and 4 illustrate the agreement between the coupled solution and the reentry data. The agreement is

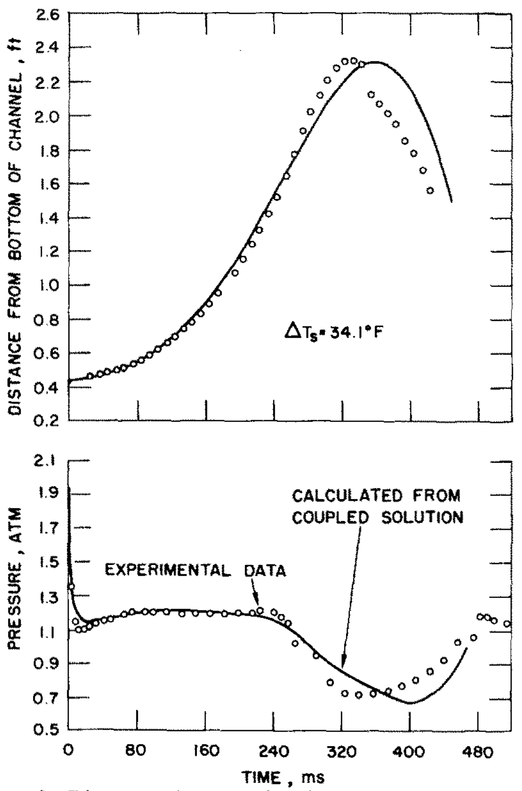

FIG. 3. Distance of upper liquid–vapor interface from bottom of channel and pressure within the vapor space vs. time for a slug expulsion and reentry by a wire electrode.

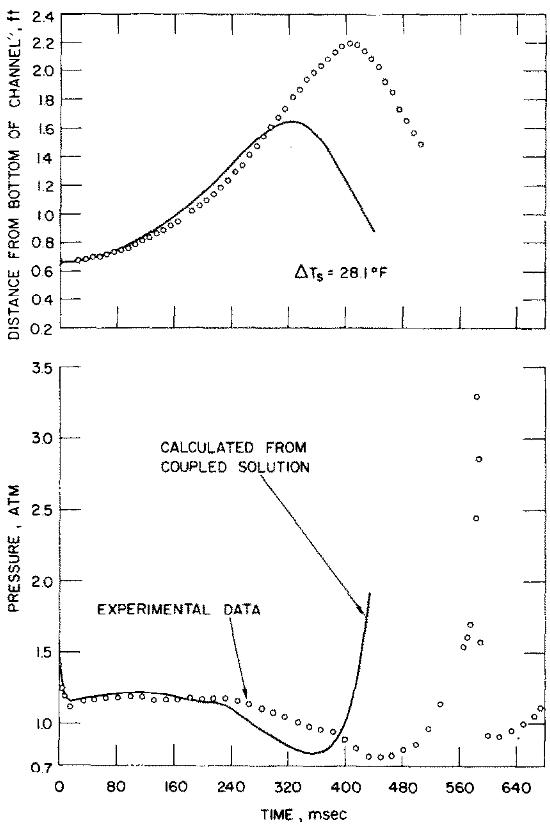

FIG. 4. Distance of upper liquid–vapor interface from bottom of channel and pressure within the vapor space vs. time for a slug expulsion and reentry by a wire electrode.

excellent for those events initiated 0·4 ft from the bottom of the channel; however, the coupled solution under-predicts by 25 per cent the point of reentry for an event initiated 0·66 ft from the bottom.

The assumption of perfectly smooth interfaces between the hot plug of liquid pushed into the cold zone and the cold film is another simplification of the actual physics of the expulsion. The mixing of the hot and cold fluids as the hot fluid passes through the cold zone has been neglected. This mixing effect will be greatest when the velocity of the liquid slug being pushed into the cold zone is small. Thus, for a low superheat expulsion initiated high in the test section as in Fig. 4, the mixing effect will be the greatest, and the coupled solution will overpredict the rate of condensation within the cold zone.

The pressure within the vapor space for the event shown in Fig. 2, in which the initial superheat is high, drops rapidly to a value intermediate between the initial pressure and the plenum pressure (0·212 atm). Thus, the growth of the vapor space for this event is both energy and intertia controlled.

Figure 3 illustrates the pressure within the vapor space for events in which the vapor slug underwent a period of growth followed by a partial collapse and additional period of growth. The pressure increases slightly during the reentry phase. As the slugs are expelled again, the pressure follows a path similar to that observed during the initial expulsion.

Figure 4 illustrates the pressure within the vapor space for an event in which total collapse occurs. A pressure increase of 3–4 atm was recorded. After the vapor space collapsed, the slight increase in pressure corresponds to bubbles nucleating from the wire electrode. The coupled solution would predict a higher pressure pulse upon reentry than that indicated above. This is due to the fact that the coupled solution does not account for compressibility effects during the reentry phase.

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CORRELATION OF LIQUID-FILM COOLING MASS TRANSFER DATA

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NOTATION

e, fraction of m_1 entrained into gas flow;
e_0, value of e for $l = l_0 = 10$ in.;
$h_{l,m}$, liquid enthalpy at $T_{l,m}$;
$h_{l,1}$, liquid enthalpy at $T_{l,1}$;
$H_{v,m}$, liquid heat of vaporization at $T_{l,m}$;
l, liquid film length;
l_0, liquid film length of 10 in.;
m_1, rate of liquid injection per unit width of film;
m, net mass-transfer rate from liquid film per unit of film width;
m'_{id}, m' for ideal case where $r = e_0 = 0$;
r, roughness parameter;
T_g, free-stream gas temperature;
T_s, liquid film surface temperature;
$T_{l,1}$, liquid temperature at point of injection;